

134 Quiz 10 Solutions

QUESTION 1

Evaluate the improper integral. $\int_e^{\infty} \frac{1}{x \ln(x)} dx =$ (Enter a number or enter "infinity")

So that improper integral is . (Enter "convergent" or "divergent")

Based on your work above, the series $\sum_{n=3}^{\infty} \frac{1}{n \ln(n)}$ is . (Enter "convergent" or "divergent")

$$\int_e^t \frac{1}{x \ln(x)} dx = \ln(\ln x) \Big|_e^t = \ln(\ln(t)) - \ln(\ln e) = \ln(\ln(t)) \xrightarrow{t \rightarrow \infty} \infty$$

integral test says $\sum \frac{1}{n \ln(n)}$ is divergent too

($f(x) = \frac{1}{x \ln(x)}$ is continuous, positive, and decreasing on $[e, \infty)$)

QUESTION 2

Determine whether the p-series is convergent or divergent.

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \dots = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

The series is a p-series with $p =$. (Enter an exact number.)

Therefore, the series is . (Enter "convergent" or "divergent")

($p \leq 1$)

QUESTION 3

Determine whether the series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{4^n - 4}{6^n + 3}$$

$$\frac{4^n - 4}{6^n + 3} \leq \frac{4^n}{6^n + 3} \leq \frac{4^n}{6^n} = \left(\frac{2}{3}\right)^n$$

$$\sum_{n=2}^{\infty} \left(\frac{2}{3}\right)^n \text{ geom. w/ } r = \frac{2}{3} < 1$$

- ☐ The series diverges by the comparison test. Each term is greater than that of a divergent geometric series.
- ☐ The series converges by the comparison test. Each term is less than that of a convergent p-series.
- ☐ The series diverges by the comparison test. Each term is greater than that of a divergent p-series.
- ☒ The series converges by the comparison test. Each term is less than that of a convergent geometric series.

QUESTION 4

Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[4]{n+1}}$$

- ☐ The series converges by the test for divergence. $\lim_{n \rightarrow \infty} \frac{(-1)^{n-1}}{\sqrt[4]{n+1}} = 0$.
this is impossible!
- ☐ The series diverges by the comparison test. Each term is greater than that of a divergent geometric series.
- ☒ The series converges by the alternating series test. The sequence $\frac{1}{\sqrt[4]{n+1}}$ decreases and converges to 0.
- ☐ The series diverges by the comparison test. Each term is greater than that of a divergent p-series.

QUESTION 5

Give an example of divergent alternating series. (Fill in the blank)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Explain (in one sentence) why your series diverges.

If $\{b_n\}$ is any sequence that does not converge to 0, then $\lim_{n \rightarrow \infty} (-1)^n b_n \neq 0$ and so $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges by the test for divergence.